The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: September 26, 2023 Course: EE 313 Evans

Name: ____
Last, First

- This in-person exam is scheduled to last 75 minutes.
- Open books, open notes, and open class materials, including homework assignments and solution sets and previous midterm exams and solutions.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- ***Please disable all wireless connections on your calculator(s) and computer system(s).***
- Please mute all computer systems.
- Please turn off all phones.
- No headphones are allowed.
- All work should be performed on the midterm exam. If more space is needed, then use the backs of the pages.
- <u>**Fully justify your answers**</u>**.** If you decide to quote text from a source, please give the quote, page number and source citation**.**

Problem	*Point Value*	*Your score*	*Topic*
1	25		Sampling Sinusoids
2	25		Fourier Series
3	26		Sampling
4	24		Spectrograms
Total	100		

Problem 1.1 *Sampling Sinusoids*. 25 points.

Consider the sinusoidal signal $x(t) = \cos\,(2\,\pi\,f_0\,t)$ for continuous-time frequency f_0 in Hz.

We are able to observe $x(t)$ for all time, i.e. for $-\infty < t < \infty$.

We sample $x(t)$ at a sampling rate of f_s in Hz to produce a discrete-time signal $x[n]$.

(a) Derive the formula for $x[n]$ by sampling $x(t)$ at a sampling rate of f_s in Hz. *6 points.*

(b) Based on your answer in part (a), give a formula for the discrete-time frequency $\hat{\omega}_0$ of $x[n]$ in terms of the continuous-time frequency f_0 and sampling rate f_s. Units of $\hat{\omega}_0$ are in rad/sample. *6 points*.

(c) We choose the sampling rate f_s to satisfy the Sampling Theorem, i.e. $f_s > 2\,f_0$.

i. Give the range of continuous-time frequencies in Hz that can be correctly captured by sampling. Be sure to include negative, positive and zero frequencies. Justify your reasoning. *6 points.*

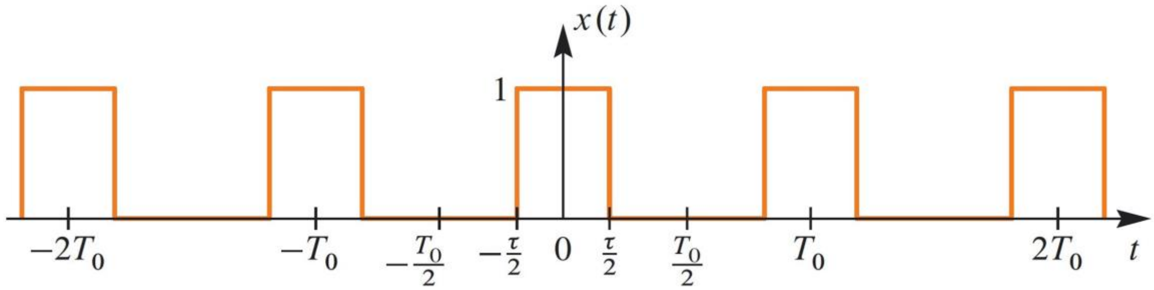
ii. Give the range of discrete-time frequencies that are represented when the Sampling Theorem is satisfied, i.e. $f_s > 2\,f_0$. Be sure to include negative, positive and zero frequencies. Justify your reasoning. *7 points*

Problem 1.2 *Fourier Series.* 25 points.

Compute the Fourier series for a periodic pulse wave

$$x(t) = \begin{cases} 1 & \text{for } 0 \le |t| < \tau/2 \\ 0 & \text{for } \tau/2 \le |t| < T_0/2 \end{cases}$$

The fraction of the time the pulse is "on" (i.e. has value 1) in each fundamental period T_0 is τ/T_0.



(a) Compute Fourier series coefficients a_k of $x(t)$ to represent $x(t) = \sum_{k=-\infty}^{\infty} a_k\, e^{j2\pi k f_0 t}$

i. Compute a_0. *6 points*. $a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\, dt$

ii. Compute a_k for $k \neq 0$. *12 points*. $a_k = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j2\pi k f_0 t}\, dt$

(b) Compute Fourier series coefficients b_k of $y(t) = 2\,x(t) - 2\,\tau/T_0$ to represent $y(t) = \sum_{k=-\infty}^{\infty} b_k\, e^{j2\pi k f_0 t}$

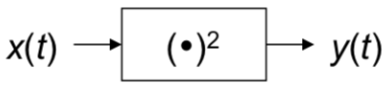
i. Compute b_0. *3 points*. $b_0 = \frac{1}{T_0}\int_0^{T_0} y(t)\, dt$

ii. Compute b_k for $k \neq 0$. *4 points*. $b_k = \frac{1}{T_0}\int_0^{T_0} y(t)\, e^{-j2\pi k f_0 t}\, dt$

Problem 1.3. *Sampling.* 26 points**.**

(a) Let $x(t) = \cos(2\pi f_0 t)$ be a continuous-time signal for $-\infty < t < \infty$.

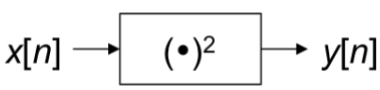
i. From the block diagram below, derive a formula for $y(t)$ and write it as a sum of cosines. *6 points*.



ii. Let $f_0 = 3000\ Hz$. What negative, zero, and positive frequencies are present in $y(t)$? *6 points*

(b) Let $x(t) = \cos(2\pi f_0 t)$ be a continuous-time signal for $-\infty < t < \infty$.

i. From the block diagram below, derive a formula for $y[n]$ and write it as a sum of cosines. *6 points*



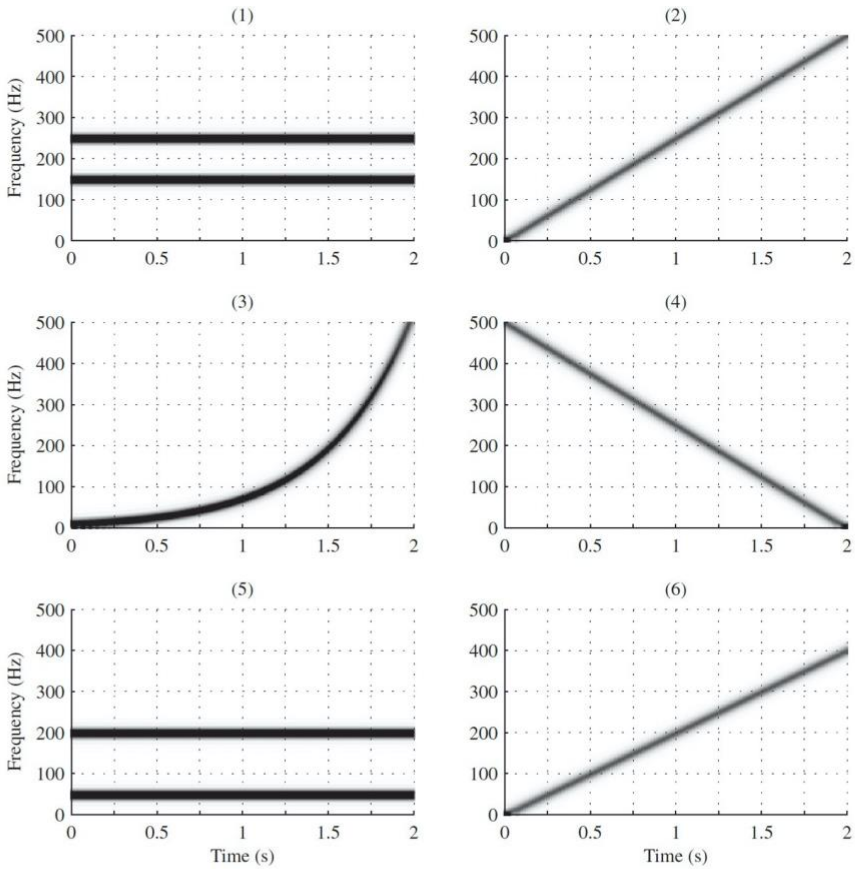
ii. Let $f_0 = 3000$ Hz and $f_s = 8000$ Hz. What negative, zero and positive discrete-time frequencies are present in $y[n]$ between –π rad/sample and π rad/sample? What are their corresponding continuous-time frequencies? *8 points*.

Problem 1.4. *Spectrograms*. 24 points.

Below are spectrograms (labeled with numbers 1-6) for six signals over the time interval $0 \le t \le 2s$.

The vertical frequency axis for each spectrogram shows non-negative frequencies in units of Hz.

For each signal (a)-(f), identify the corresponding spectrogram and explain your reasoning.



(a) $x(t) = \cos(-250\pi\, t^2)$

(b) $x(t) = \cos(100\pi\, t - \pi/4) + \cos(400\pi\, t)$

(c) $x(t) = \cos(1000\pi\, t - 250\pi\, t^2)$

(d) $x(t) = \cos(100\pi\, t)\cos(400\pi\, t)$

(e) $x(t) = \cos(30e^{2t})$

(f) $x(t) = \cos(200\pi\, t^2)$